

Supersymmetric electric-magnetic duality as a manifest symmetry of the action for super-Maxwell theory and linearized supergravity

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Abstract

For the free massless spin-one and spin-two field theories one may write the action in a form which is manifestly invariant under electric-magnetic duality. This is achieved by introducing new potentials through solving the constraints of the Hamiltonian formulation. The price for making electric-magnetic duality invariance manifest through this direct procedure is losing manifest Lorentz invariance. Both theories admit supersymmetric extensions, which make the bosonic fields and their corresponding fermionic partners to be parts of the same geometrical object, a supermultiplet. We present in this paper the supersymmetric extension of the manifestly electric-magnetic duality invariant actions for the photon and the photino; and for the graviton and the gravitino. In each case the spinor fields transform under electric-magnetic duality in a chiral manner. For the spin-tree-half field, which possesses a gauge invariance, it is necessary to bring in a spinor “pre-potential”. As in previous cases the introduction of additional potentials to solve the constraints increases the number of gauge invariances of the action, thus keeping the number of degrees of freedom unaltered. The similarity in the formulations for the photon-photino and graviton-gravitino systems is remarkable.

1 Introduction

Electric-magnetic duality is a fascinating symmetry that keeps appearing in many contexts. It was shown long ago [1] that for Maxwell theory it can be implemented as a manifest symmetry of the action in terms of *two* vector potentials, the ordinary one for the magnetic field and an additional one for the electric field. The second potential was introduced by solving the constraint (Gauss' law) of the Hamiltonian formulation. The price for achieving manifest electric-magnetic duality invariance through this direct procedure is losing manifest Lorentz invariance.

The approach to electric-magnetic duality based on solving the Hamiltonian constraints has been fruitfully applied later on to p -forms, including Chern-Simons couplings and coupling to scalar fields defined on a coset space [2, 3, 4]. Furthermore it has been shown [5] that linearized Einstein gravity can also be formulated through this procedure in a manner which makes the action manifestly invariant under electric-magnetic duality transformations. That result was achieved by solving the Hamiltonian and momentum constraints in terms of two “prepotentials” which are two-index symmetric tensors. In all cases the introduction of additional potentials to solve the constraints increases the number of gauge invariances of the action, thus keeping the number of physical degrees of freedom unaltered.

We consider in this paper the supersymmetric extensions of Maxwell theory and linearized Einstein gravity, i.e., super-Maxwell theory and linearized supergravity. We provide a formulation in which electric-magnetic duality is a manifest symmetry of the action for both the photon-photino and graviton-gravitino systems. For the gravitino one must introduce a new spinor “prepotential” which is the fermionic superpartner of the symmetric tensor bosonic prepotentials previously found in linearized gravity. For the photino, since there are no fermionic gauge symmetries, and thus no fermionic constraints to solve, the original spinor field may be regarded as its own prepotential. In order to properly implement electric-magnetic duality together with supersymmetry it is necessary to define the former as acting chirally on the spinors. (The interplay between chirality and duality invariance was particularly stressed in [6].) The very close parallel between the formulation for the photon-photino case and that for the graviton-gravitino case is quite remarkable.

The paper is organized as follows: Section 2 reviews the supersymmetric extension of the standard one-potential Maxwell theory. The supersymmetric

extension of the manifestly electric-magnetic duality invariant two-potential formulation is then derived. It is shown that if one demands that electric-magnetic duality and supersymmetry commute, then the duality transformation of the photino is a chiral rotation. Section 3 employs the insights obtained in section 2 to develop the corresponding formulation for linearized supergravity. Since the key results are remarkably similar to those of section 2, their proof is relegated to the Appendices in order not to deviate the attention of the reader from the main line of argument. Finally section 4 is devoted to concluding remarks. We work in four-dimensional Minkowski space throughout.

2 Electric-Magnetic Duality for the Photon and the Photino

2.1 Two-potential formulation of the Maxwell theory

If, besides the standard “magnetic” vector potential defined through,

$$\vec{B} \equiv \vec{B}_1 = \vec{\nabla} \times \vec{A}_1,$$

one introduces an additional vector potential \vec{A}_2 through,

$$\vec{E} \equiv \vec{B}_2 = \vec{\nabla} \times \vec{A}_2,$$

one may rewrite the standard Maxwell action

$$I = -\frac{1}{4} \int d^4x F^{\mu\nu} F_{\mu\nu} \quad (2.1)$$

with,

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu,$$

in terms of the two potentials A^a as [1] (see also [7])

$$I = \frac{1}{2} \int dx^0 d^3x \left(\epsilon_{ab} \vec{B}^a \cdot \dot{\vec{A}}^b - \delta_{ab} \vec{B}^a \cdot \vec{B}^b \right). \quad (2.2)$$

Here, ϵ_{ab} is given by $\epsilon_{ab} = -\epsilon_{ba}$, $\epsilon_{12} = +1$, or as a matrix

$$\epsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \epsilon^2 = -I \quad (2.3)$$

where I is the 2 by 2 unit matrix.

The action (2.2) is invariant under rotations in the $(1, 2)$ plane (“electric-magnetic duality rotations”) ,

$$\begin{pmatrix} \vec{A}^1 \\ \vec{A}^2 \end{pmatrix} \equiv \vec{\mathbf{A}} \longrightarrow e^{\alpha\epsilon} \vec{\mathbf{A}} \quad (2.4)$$

because ϵ_{ab} and δ_{ab} are invariant tensors. Note that ϵ given by (2.3) generates *clockwise rotations*. This convention makes the formulas below more symmetric. The action (2.2) is also invariant under the gauge transformations,

$$\vec{A}^a \longrightarrow \vec{A}^a + \vec{\nabla} \Lambda^a.$$

2.2 Supersymmetric extension of the two-potential theory

The action for the free supermultiplet formed by the spin one photon and the spin one-half photino has the form,

$$I = I_{\text{BOSE}} + I_{\text{FERMI}}, \quad (2.5)$$

where I_{BOSE} is the action for the photon and I_{FERMI} is given by,

$$I_{\text{FERMI}} = -\frac{i}{2} \int d^4x \bar{\psi} \gamma^\mu \partial_\mu \psi$$

where ψ is an anticommuting Majorana spinor. The action I_{FERMI} is invariant under the chirality transformation,

$$\psi \longrightarrow e^{\beta\gamma_5} \psi,$$

which is an $SO(2)$ -rotation because,

$$(\gamma_5)^2 = -I.$$

The matrix I is here the 4 by 4 unit matrix.

If one takes for I_{BOSE} in (2.5) the standard one-potential action (2.1), the sum $I_{\text{BOSE}} + I_{\text{FERMI}}$ is invariant under the infinitesimal supersymmetry transformations

$$\delta A_\mu = i\bar{\epsilon} \gamma_\mu \psi, \quad (2.6)$$

$$\delta \psi = \frac{1}{2} \gamma^{\mu\nu} F_{\mu\nu} \epsilon, \quad (2.7)$$

where ϵ is a constant anticommuting Majorana spinor. [See [8] for a lucid presentation.] In order to determine the realization of the supersymmetry transformation in the two-potential theory, we observe that starting from it one can go to a purely electric or a purely magnetic representation by eliminating either the electric or the magnetic potential respectively. (We follow standard terminology: the electric representation is the one where the electric charge couples to the usual “magnetic” vector potential.) In the magnetic representation, the Maxwell action takes the form (2.1) with $F_{\mu\nu}$ replaced by the field strength of the second vector potential A_μ^2 ,

$$F_{\mu\nu}^2 = \partial_\mu A_\nu^2 - \partial_\nu A_\mu^2.$$

Therefore, the action is invariant under the transformation

$$\delta A_\mu^2 = i\bar{\eta}\gamma_\mu\psi, \quad (2.8)$$

$$\delta\psi = \frac{1}{2}\gamma^{\mu\nu} F_{\mu\nu}^2 \eta, \quad (2.9)$$

for some spinor η . To determine the relation between η and ϵ , one recalls that on-shell one has, $F_{\mu\nu}^2 = {}^*F_{\mu\nu} \equiv \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}$, and one uses the identity ${}^*\gamma_{\mu\nu} = \gamma_5\gamma_{\mu\nu}$. This gives $\eta = -\gamma_5\epsilon$. Thus the supersymmetry transformation in the magnetic representation is given by

$$\delta A_\mu^2 = i\bar{\epsilon}\gamma_\mu\gamma_5\psi. \quad (2.10)$$

$$\delta\psi = -\frac{1}{2}\gamma^{\mu\nu} F_{\mu\nu}^2 \gamma_5\epsilon, \quad (2.11)$$

The presence of γ_5 makes δA_μ^2 a pseudo-vector, as it should be the case since A_μ^2 is the potential for ${}^*F_{\mu\nu}$.

To pass to the two-potential representation, we may start from the electric representation and expand the sum over $[\mu\nu]$ on the right-hand side of (2.7) into electric $[0i]$ and magnetic $[mn]$ pieces. Inserting the expression for the electric and magnetic fields as curls of their respective potentials, we obtain,

$$\delta\psi = \gamma^{rs}\partial_r (A_s^1 - \gamma_5 A_s^2). \quad (2.12)$$

This expression, together with the spatial parts of (2.6) and (2.10),

$$\delta\vec{\mathbf{A}} = \delta \begin{pmatrix} \vec{A}^1 \\ \vec{A}^2 \end{pmatrix} = i\bar{\epsilon}\vec{\gamma} \begin{pmatrix} 1 \\ \gamma_5 \end{pmatrix} \psi, \quad (2.13)$$

is the the supersymmetry transformation, which leaves invariant the action (2.5) with I_{BOSE} given by (2.2).

Now we pass to discuss the interplay between supersymmetry and duality. To that effect we observe that under an electric-magnetic duality transformation of the form (2.4) on \vec{A}^a , one has,

$$\vec{A}^1 - \gamma_5 \vec{A}^2 \longrightarrow e^{\alpha\gamma_5} (\vec{A}^1 - \gamma_5 \vec{A}^2).$$

Then $\delta\psi$ in (2.12) transforms as,

$$\delta\psi \longrightarrow e^{\alpha\gamma_5} \delta\psi.$$

Conversely, if one transforms ψ as,

$$\psi \longrightarrow e^{\alpha\gamma_5} \psi,$$

then $\delta\vec{A}$ in (2.13) transforms according to a duality rotation,

$$\delta\vec{A} \longrightarrow e^{\alpha\epsilon} \delta\vec{A}.$$

This implies that *supersymmetry and electric-magnetic duality commute* if one defines the latter as a transformation of *both the vectors \vec{A}^a and the spinor ψ* ,

$$\vec{A} \longrightarrow e^{\alpha\epsilon} \vec{A}, \quad \psi \longrightarrow e^{\alpha\gamma_5} \psi. \quad (2.14)$$

The parallelism between these two transformations becomes all the more poignant if one uses the Majorana representation given in the Appendix A. Then the Majorana spinors are real and

$$\gamma_5 = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}.$$

On the other hand, as given above,

$$\epsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$

Actually ϵ can be thought of as a 4×4 matrix when acting on the four independent components of the gauge invariant curl $\vec{\nabla} \times \vec{A}$. It then becomes γ_5 itself!

The beautiful similarity between the duality transformation properties of the vector and spinor fields arose because although uncoupled they were related by supersymmetry.

The supersymmetry transformation of the non-manifestly Lorentz invariant two potential formulation of the super-Maxwell theory can also be obtained from the results of [9] on the manifestly $E_{7,7}$ -invariant formulation of maximal supergravity in four space-time dimensions by truncating the corresponding formulas to a subsector of the theory containing only one vector field and one of its supersymmetric spin 1/2 partner. We have provided above a different, direct derivation of the supersymmetry transformations using the existence of the “electric” and “magnetic” representations.

3 Electric-Magnetic Duality for the Graviton and the Gravitino

We will develop in this section the formulation of linearized supergravity in terms of prepotentials, which makes electric-magnetic duality to be a manifest invariance of the action. The corresponding formulation for linearized gravity alone was given in [5]. In that case the solution of the Hamiltonian and momentum constraints, which generate the Hamiltonian version of linearized spacetime diffeomorphisms, led to the introduction of two symmetric tensor prepotentials for the canonical pair formed by the spatial metric and its canonical conjugate, the extrinsic curvature.

Supergravity brings in an additional fermionic gauge invariance and, with it, an additional Majorana spinor constraint that generates it. That constraint can be solved to express the canonically self-conjugate vector-spinor gauge potential of the theory in terms of a vector-spinor prepotential.

It was natural to expect, by analogy with the case of the photon and the photino, that a similar formulation for the graviton and the gravitino, which exhibits manifest electric-magnetic duality invariance, could be achieved by focusing on the bosonic and fermionic prepotentials and their transformation properties under global supersymmetry. This not only turned out to be the case, but the result transcended the most optimistic expectations: the ensuing formulation for the spin two/spin three-half multiplet is not just similar but *identical* to the one for the spin one/spin one-half case. For that reason, we will only give the results in the main text and provide the proofs

of the key equations in the Appendices.

3.1 Prepotentials for the graviton and the gravitino

For linearized gravity the fields appearing in the canonical formulation are the linearized spatial metric h_{ij} ($g_{ij} = \delta_{ij} + h_{ij}$) and its canonically conjugate momentum π^{ij} . They obey the Hamiltonian and momentum constraints,

$$\begin{aligned}\partial^m \partial^n h_{mn} - \triangle h &= 0, \\ \pi^{mn}{}_{,n} &= 0,\end{aligned}$$

where $h = h^m_m$ and \triangle is the Laplacian.

One may solve [5] these constraints by introducing two prepotentials Z^a_{mn} such that,

$$\begin{aligned}\pi^{mn} &= \epsilon^{mpq} \epsilon^{nrs} \partial_p \partial_r Z^1_{qs}, \\ h_{mn} &= \epsilon_m{}^{rs} \partial_r Z^2_{sn} + \epsilon_n{}^{rs} \partial_r Z^2_{sm}.\end{aligned}$$

Note that Z^1 transforms as a scalar under inversions, whereas Z^2 transforms as a pseudo-scalar, just as in the Maxwell case. Given h_{mn} and π^{mn} up to a gauge transformation, there are ambiguities

$$Z^a_{mn} \longrightarrow Z^a_{mn} + \partial_m \xi^a_n + \partial_n \xi^a_m + \xi^a \delta_{mn}, \quad (3.1)$$

which constitute the gauge transformations of the prepotentials. The terms with ξ^a_m are linearized diffeomorphisms, while the last term takes the form of a linearized Weyl rescaling.

In linearized supergravity there is, in addition to the conjugate pair (h_{mn}, π^{mn}) , the self-conjugate Majorana vector-spinor ψ_m , which is an anti-commuting field that has four real components in a Majorana representation, in which the Dirac matrices are real.

The vector-spinor ψ_m obeys the constraint

$$\gamma^{mn} \partial_m \psi_n = 0 \quad (3.2)$$

which may be solved (see Appendix B.1) in terms of another vector-spinor χ_p as,

$$\psi_m = \gamma_5 (\delta_{ms} - \gamma_{ms}) \epsilon^{skp} \partial_k \chi_p. \quad (3.3)$$

Given ψ_m up to a gauge transformation $\psi_m \rightarrow \psi_m + \partial_m \epsilon$, there are the ambiguities,

$$\chi_p \longrightarrow \chi_p + \partial_p \eta + \gamma_p \rho, \quad (3.4)$$

where η, ρ are Majorana spinors. These constitute the gauge symmetries for the fermionic prepotential χ_p . Note the close resemblance of the transformations for the twelve Z_{mn}^a and the twelve χ_m^A .

It is noteworthy that, while the gauge transformations of the bosonic prepotentials are those of linearized conformal gravity, the gauge transformations (3.4) of the fermionic prepotential turn out to be exactly those of linearized conformal supergravity, with two independent fermionic symmetries [10], parametrized here by the two independent spinor parameters η and ρ .

3.2 Supersymmetry transformations of the prepotentials

First, we recall how to infer the global supersymmetry transformations of the free graviton and gravitino fields $h_{\mu\nu}$ and ψ_μ^A . Starting from them we will obtain the corresponding transformations for the prepotentials Z_{mn}^a and χ_m^A . Actually, the procedure is nothing but undoing the steps that originally led to supergravity through gauging the global symmetries of the free spin-two/spin-three-half theory. We follow this “reversed route” because it brings in immediately the geometrical quantities that we need, i.e., the spatial metric and connection, and the extrinsic curvature.

One starts by imagining expanding the action for full supergravity in powers of the fields and their derivatives as $I^{(2)} + I^{(3)} + \dots$ where $I^{(k)}$ is of k -th polynomial degree. The quadratic part $I^{(2)}$ is the action for linearized supergravity and is the sum of the free spin-2 and free spin-3/2 actions. Similarly, one can expand the supersymmetry transformations under which the full action is invariant as $\delta_\epsilon \Phi^i = \delta_\epsilon^{(1)} \Phi^i + \delta_\epsilon^{(2)} \Phi^i + \dots$ where the expansion is again performed according to the polynomial degree, counting also the gauge parameter ϵ . Here, Φ^i stands for all the fields. Explicitly,

$$\delta_\epsilon^{(1)} h_{\mu\nu} = 0, \quad \delta_\epsilon^{(2)} h_{\mu\nu} = i (\bar{\epsilon} \gamma_\mu \psi_\nu + \bar{\epsilon} \gamma_\nu \psi_\mu) \quad (3.5)$$

for the graviton and

$$\delta_\epsilon^{(1)} \psi_\mu = 4 \partial_\mu \epsilon, \quad \delta_\epsilon^{(2)} \psi_\mu = -\omega_{\mu\rho\sigma} \gamma^{\rho\sigma} \epsilon, \quad (3.6)$$

for the gravitino. Here, $\omega_{\mu\rho\sigma}$ is the linearized spin connection (see appendix A) so that $\delta_\epsilon^{(1)}\psi_\mu + \delta_\epsilon^{(2)}\psi_\mu = 4D_\mu\epsilon$. (We have chosen a normalization so that the formulas for the prepotentials look simple.)

The invariance of the full action under the full supersymmetry transformations implies, when expanded according to the polynomial degree, a chain of equations: $\delta_\epsilon^{(1)}I^{(2)} = 0$ (lowest order), $\delta_\epsilon^{(2)}I^{(2)} + \delta_\epsilon^{(1)}I^{(3)} = 0$ (next order), etc. The first of these equations just expresses the invariance of the action $I^{(2)}$ of linearized supergravity (free Rarita-Schwinger action) under the abelian gauge supersymmetry transformations $\delta_\epsilon^{(1)}\Phi^i$. As we have just recalled above, these transformations reduce to $\delta_\epsilon^{(1)}\psi_\mu = 4\partial_\mu\epsilon$, $\delta_\epsilon^{(1)}h_{\mu\nu} = 0$.

Since $\delta_\epsilon^{(1)}\Phi^i$ contains only derivatives of ϵ , it is identically equal to zero for constant (i.e., spacetime independent) ϵ 's. Therefore, $\delta_\epsilon^{(1)}I^{(k)} \equiv 0$ for all k 's, and in particular $\delta_\epsilon^{(1)}I^{(3)} \equiv 0$, when ϵ is taken to be a constant. It follows from $\delta_\epsilon^{(2)}I^{(2)} + \delta_\epsilon^{(1)}I^{(3)} = 0$ that the action $I^{(2)}$ of linearized supergravity is invariant under the rigid supersymmetry transformations $\delta_\epsilon^{(2)}\Phi^i$ with constant ϵ 's, $\delta_\epsilon^{(2)}I^{(2)} = 0$. So, in addition to the gauge invariance under the local supersymmetry transformations $\delta_\epsilon^{(1)}\Phi^i$ (with ϵ an arbitrary spacetime dependent spinor), the free action of linearized supergravity possesses also the invariance under the “rigid supersymmetry” $\delta_\epsilon^{(2)}\Phi^i$ with constant ϵ . It is this latter transformation that we want to write for the prepotentials.

The derivation is given in Appendix C. One finds, starting from (3.5) and (3.6) with constant ϵ ,

$$\delta\chi_p = \gamma^{rs}\partial_s (Z_{rp}^1 - \gamma_5 Z_{rp}^2) \epsilon, \quad (3.7)$$

and,

$$\delta\mathbf{Z}_{mn} = \delta \begin{pmatrix} Z_{mn}^1 \\ Z_{mn}^2 \end{pmatrix} = i\bar{\epsilon} \left(\gamma_m \begin{pmatrix} 1 \\ \gamma_5 \end{pmatrix} \chi_n + \gamma_n \begin{pmatrix} 1 \\ \gamma_5 \end{pmatrix} \chi_m \right). \quad (3.8)$$

Equations (3.7) and (3.8) are the same as (2.12) and (2.13) with \mathbf{A}_m replaced by \mathbf{Z}_{mn} and χ replaced by χ_p . Therefore, all the conclusions of Subsection 2.2 translate literally. Electric-magnetic duality acts on both \mathbf{Z} and χ as,

$$\mathbf{Z} \longrightarrow e^{\alpha\epsilon} \mathbf{Z}; \quad \chi \longrightarrow e^{\alpha\gamma_5} \chi,$$

and it commutes with supersymmetry.

Note in this context that, unlike the photon-photino case, if the Z_{sp}^a in (3.7) undergo a gauge transformation, the right hand side of that equation experiences a transformation of the form (3.4). Similarly for (3.8). This

phenomenon does not have an analog for the spin one - spin one-half case because there the spin one-half field is its own prepotential and does not possess gauge freedom.

3.3 Manifestly duality invariant action

To complete the presentation for the graviton case, we present here the action expressed in terms of prepotentials.

The action is,

$$I = I_{\text{BOSE}} + I_{\text{FERMI}},$$

where I_{BOSE} was given in [5],

$$S[Z_a^{mn}] = \int dt \left[\int d^3x \epsilon^{ab} \epsilon^{mrs} (\partial^p \partial^q \partial_r Z_{aps} - \Delta \partial_r Z_a^q{}_s) \dot{Z}_{bqm} - H_{\text{BOSE}} \right], \quad (3.9)$$

with,

$$\begin{aligned} H_{\text{BOSE}} = & \int d^3x \delta^{ab} \left(\Delta Z_{aij} \Delta Z_b^{ij} + \frac{1}{2} \partial^k \partial^m Z_{akm} \partial^q \partial^n Z_{bqn} + \partial^k \partial^m Z_{akm} \Delta Z_b \right) \\ & + \int d^3x \delta^{ab} \left(-2 \partial_m \partial_i Z_a^{ij} \partial^m \partial^k Z_{bkj} - \frac{1}{2} \Delta Z_a \Delta Z_b \right), \end{aligned} \quad (3.10)$$

and $Z^a = Z^{am}{}_m$. This action is manifestly invariant under electric-magnetic duality because ϵ_{ab} and δ_{ab} are invariant tensors. A more transparent rewriting of the action I_{BOSE} in which its gauge invariance under linearized diffeomorphisms and Weyl rescalings are made more explicit is given in [11].

The fermionic action is obtained by inserting in the Rarita-Schwinger action the expression (3.3) for the original vector-spinor ψ_m in terms of its prepotential χ_p . The component ψ_0 drops out because the spinor constraint vanishes identically in terms of the prepotential.

One finds (see Appendix B.2),

$$I_{\text{FERMI}} = \int dx^0 \left(-i \int d^3x \bar{\Sigma}_{mn} \gamma^{mnp} \dot{\chi}_p - H_{\text{FERMI}} \right),$$

with

$$H_{\text{FERMI}} = -\frac{i}{4} \int d^3x \bar{\Sigma}^{mn} \gamma^0 \gamma_5 (\delta_{mk} - 2\gamma_{mk}) \Delta_n^k.$$

Here the tensor-spinor field strengths Δ_{pq} and Σ_{pq} are respectively defined by

$$\Delta_{pq} = \partial_p \chi_q - \partial_q \chi_p,$$

and

$$\Sigma_{pq} = \frac{1}{2} \gamma_5 (\delta_{qs} - \gamma_{qs}) \epsilon^{skm} \partial_p \Delta_{km} - (p \leftrightarrow q).$$

The tensor-spinor Σ_{pq} is gauge invariant under all gauge symmetries of χ_p and fulfills the identity $\Sigma_{pq} \gamma^{pq} = 0$, in addition to $\partial_{[r} \Sigma_{pq]} = 0$.

The action I_{FERMI} is invariant under the chirality transformation $\chi_p \rightarrow e^{\alpha \gamma_5} \chi_p$ and is therefore electric-magnetic duality invariant.

4 Concluding Remarks

In this paper, we have derived the manifestly electric-magnetic duality invariant formulation of super-Maxwell theory and linearized supergravity. The manifestly duality invariant action of super-Maxwell theory involves two potentials, while the manifestly duality invariant action of linearized supergravity involves one fermionic prepotential besides the two bosonic prepotentials of linearized gravity [5]. The supersymmetry transformations of the two potentials and the spin-one-half field (super-Maxwell theory) and of the three prepotentials (linearized supergravity) have been explicitly written and are local. They take a remarkably similar form in both cases.

Supergravity and electric-magnetic duality have a well known fruitful interplay [12]. But we stress that the duality discussed here in the context of linearized supergravity is a gravitational duality that acts on the graviton. It is present already for the $N = 1$ theory (and for that matter, even for $N = 0$), without the vector or scalar fields present in the extended models and on which duality is traditionally considered.

It would be of interest to include a cosmological constant in our analysis and to analyze gauged supergravities. Existing work in that direction is encouraging [13] (see also [14] for related comments).

Finally, the asymptotic properties of the manifestly duality invariant formulation of supergravity are also of definite interest, given the enlargement of the gauge group. This problem is currently under study (see [15] in that context).

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Appendices

A Conventions - γ -matrices

The Dirac γ -matrices fulfill,

$$\gamma_\mu \gamma_\nu + \gamma_\nu \gamma_\mu = 2\eta_{\mu\nu}$$

where $\eta_{\mu\nu}$ has the “mostly +” signature $(-, +, +, +)$.

We adopt a Majorana representation where the γ -matrices are real, with antisymmetric γ_0 and symmetric γ_k 's,

$$(\gamma_0)^T = -\gamma_0, \quad (\gamma_k)^T = \gamma_k.$$

The matrix γ_5 is defined through,

$$\gamma_5 = \gamma_0 \gamma_1 \gamma_2 \gamma_3,$$

and fulfills,

$$(\gamma_5)^2 = -I, \quad (\gamma_5)^T = -\gamma_5.$$

We define $\gamma_{\mu\nu} = \frac{1}{2}(\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu)$ and $\gamma_{\mu\nu\rho} = \frac{1}{3!}(\gamma_\mu \gamma_\nu \gamma_\rho + \gamma_\nu \gamma_\rho \gamma_\mu + \gamma_\rho \gamma_\mu \gamma_\nu - \gamma_\mu \gamma_\rho \gamma_\nu - \gamma_\rho \gamma_\nu \gamma_\mu - \gamma_\nu \gamma_\mu \gamma_\rho) = \epsilon_{\mu\nu\rho\sigma} \gamma_5 \gamma^\sigma$ with $\epsilon_{0123} = +1$.

Useful relations are:

$$\begin{aligned} \epsilon^{kmp} \gamma_p &= \gamma^{km} \gamma_0 \gamma_5, \\ \gamma^{mn} \gamma^s &= -\epsilon^{mns} \gamma_0 \gamma_5 + \delta^{ns} \gamma^m - \delta^{ms} \gamma^n, \\ \gamma^{mn} \gamma^{rs} &= \delta^{nr} \gamma^{ms} - \delta^{ns} \gamma^{mr} - \delta^{mr} \gamma^{ns} + \delta^{ms} \gamma^{nr} + \delta^{nr} \delta^{ms} - \delta^{mr} \delta^{ns}, \\ \gamma^{mn} \gamma_{np} &= 2\delta_p^m + \gamma_p^m, \\ \gamma^{mn} \gamma^{0s} &= -\epsilon^{mns} \gamma_5 + \delta^{ns} \gamma^{0m} - \delta^{ms} \gamma^{0n}, \\ \gamma^{abc} &= \epsilon^{abc} \gamma^0 \gamma_5, \\ {}^* \gamma^{\mu\nu} &= \gamma_5 \gamma^{\mu\nu}. \end{aligned}$$

An explicit Majorana representation of the γ -matrices is given by,

$$\gamma_0 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \quad \gamma_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

$$\gamma_2 = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}, \quad \gamma_3 = \begin{pmatrix} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix},$$

$$\gamma_5 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}.$$

The Dirac adjoint is defined by

$$\bar{\psi} = \psi^T \gamma_0$$

The covariant derivatives of a spinor field is,

$$D_\mu \psi = \partial_\mu \psi - \frac{1}{4} \omega_{\mu\rho\sigma} \gamma^{\rho\sigma} \psi.$$

The spin connection in the linearized theory is,

$$\omega_{\mu\rho\sigma} = \frac{1}{2} (\partial_\rho h_{\mu\sigma} - \partial_\sigma h_{\mu\rho}). \quad (\text{A.1})$$

B Action for the Rarita-Schwinger field

The one-potential action for the Rarita Schwinger field is,

$$I_{\text{FERMI}} = -\frac{i}{2} \int d^4x \bar{\psi}_\alpha \gamma^{\alpha\beta\gamma} \partial_\beta \psi_\gamma.$$

The equations of motion for the spin $\frac{3}{2}$ field are,

$$\gamma^{\alpha\beta\gamma} (\partial_\beta \psi_\gamma - \partial_\gamma \psi_\beta) = 0.$$

For $\alpha = 0$, one obtains the constraint

$$\gamma^{mn} \partial_m \psi_n = 0, \quad (\text{B.1})$$

whereas for $\alpha = a$,

$$\gamma^{ac} (\partial_0 \psi_c - \partial_c \psi_0) = \epsilon^{abc} \gamma_5 \partial_b \psi_c. \quad (\text{B.2})$$

One can solve for $\partial_0\psi_a$,

$$\partial_0\psi_a = \partial_a\psi_0 - \frac{1}{2}\epsilon_{abc}\gamma_5\partial^b\psi^c + \frac{1}{2}\gamma_{am}\epsilon^{mbc}\gamma_5\partial_b\psi_c. \quad (\text{B.3})$$

We also recall the expression of the linearized extrinsic curvature,

$$K_{ij} = -\frac{1}{2}(\partial_0 h_{ij} - \partial_i h_{0j} - \partial_j h_{0i}), \quad (\text{B.4})$$

and that the “first Hamiltonian equation of motion” for the spin-2 field may be written ,

$$\pi^{ij} = -K^{ij} + K\delta^{ij}.$$

B.1 Solving the constraint for ψ_k

The constraint (B.1) may be rewritten as,

$$\gamma^{mn}\psi_n = 2\partial_k(\epsilon^{kmp}\gamma_5\chi_p),$$

for some “prepotential” χ_p , which is also a vector-spinor. One can explicitly express ψ_k in terms of χ_k ,

$$\psi_m = \gamma_5(\delta_{ms} - \gamma_{ms})\epsilon^{skp}\partial_k\chi_p. \quad (\text{B.5})$$

Given ψ_p up to a gauge transformation $\psi_p \rightarrow \psi_p + \partial_p\varepsilon$, χ_p is determined up to

$$\chi_p \rightarrow \chi_p + \partial_p\eta + \gamma_p\rho, \quad (\text{B.6})$$

where η and ρ are spinor fields. Note that $\varepsilon = 2\gamma_0\rho$ and that ψ_m is invariant if $\chi_p \rightarrow \chi_p + \partial_p\eta$.

As stressed in the main text, it is remarkable that (B.6) are just the linearized fermionic gauge transformations of conformal gravity which, as it is well known, has two independent supersymmetries (“ Q ” and “ S ”) [10].

B.2 Action of linearized supergravity in terms of the prepotentials

To write the Rarita-Schwinger action in terms of the vector-spinor χ_p , we first introduce some tensor-spinors. These are the field strength of χ_p ,

$$\Delta_{pq} = \partial_p\chi_q - \partial_q\chi_p$$

and the field strength of ψ_p ,

$$\Sigma_{pq} = \partial_p \psi_q - \partial_q \psi_p$$

where ψ_m is the function of χ_p defined by (B.5), which one may rewrite as,

$$\psi_m = \frac{1}{2} \gamma_5 (\delta_{ms} - \gamma_{ms}) \epsilon^{skp} \Delta_{kp}.$$

The field strength Δ_{pq} is invariant under the η -gauge symmetry in (B.6) but not the ρ -one, while the tensor-spinor Σ_{pq} is invariant under both. Furthermore, Σ_{pq} fulfills the identity,

$$\gamma^{pq} \Sigma_{pq} = 0.$$

If one replaces in the Rarita-Schwinger action the vector-spinor ψ_p by its expression (B.5) in terms of χ_p , one gets,

$$I_{\text{FERMI}} = \int dx^0 \left(-i \int d^3x \bar{\Sigma}_{mn} \gamma^{mnp} \dot{\chi}_p - H_{\text{FERMI}} \right),$$

with

$$H_{\text{FERMI}} = -\frac{i}{4} \int d^3x \bar{\Sigma}^{mn} \gamma^0 \gamma_5 (\delta_{mk} - 2\gamma_{mk}) \Delta_n^k.$$

C Supersymmetry transformations of the prepotentials

C.1 Fermionic prepotential χ_p

One has,

$$\delta \psi_m = 4D_m \epsilon = -\omega_{m\rho\sigma} \gamma^{\rho\sigma} \epsilon$$

for constant supersymmetry transformations, where we have kept only the terms linear in the fields. There are two terms in this equation, one involving only the spatial spin connection ω_{mrs} and the other involving the extrinsic curvature, which is proportional to ω_{m0s} , as shown by (A.1) and (B.4). Explicitly,

$$\delta \psi_m = \delta_h \psi_m + \delta_\pi \psi_m,$$

where,

$$\delta_h \psi_m = -\partial_r h_{ms} \gamma^{rs} \epsilon,$$

and

$$\delta_\pi \psi_m = -(\partial_0 h_{ms} - \partial_s h_{m0}) \gamma^{0s} \epsilon.$$

C.1.1 Term containing the prepotential Z_{mn}^1

Turn first to $\delta_\pi \psi_m$. By adding the gauge transformation $\partial_m (h_{s0} \gamma^{0s} \epsilon)$ to ψ_m , which is permissible, one can rewrite it as,

$$\delta_\pi \psi_m = 2K_{ms} \gamma^{0s} \epsilon.$$

To evaluate the corresponding variation of the spinor prepotential χ_p , we multiply this expression by γ^{mn} to get,

$$\gamma^{mn} \delta_\pi \psi_n = 2K_{ns} \gamma^{mn} \gamma^{0s} \epsilon.$$

Using the identity for $\gamma^{mn} \gamma^{0s}$ given above, this can be transformed into,

$$\gamma^{mn} \delta_\pi \psi_n = 2\pi^m{}_n \gamma^{0n} \epsilon = -2\pi^{mn} \gamma_{0n} \epsilon.$$

On the other hand, $\gamma^{mn} \delta_\pi \psi_n = 2\partial_k (\epsilon^{kmp} \gamma_5 \delta \chi_p)$, while $\pi^{mn} = \epsilon^{mkp} \epsilon^{nrs} \partial_k \partial_r P_{ps}$, so that we have,

$$\epsilon^{kmp} \partial_k (\gamma_5 \delta \chi_p - \epsilon^{nrs} \partial_r P_{ps} \gamma_{0n} \epsilon) = 0,$$

from which one infers, up to a gauge transformation $\chi_p \rightarrow \chi_p + \partial_p \eta$ that can be dropped,

$$\gamma_5 \delta \chi_p = \epsilon^{nrs} \partial_r P_{ps} \gamma_{0n} \epsilon,$$

an expression that can be rewritten as,

$$\delta \chi_p = \partial_s P_{pr} \gamma_5 \gamma^{rs} \epsilon. \quad (C.1)$$

C.1.2 Term containing the prepotential Z_{mn}^2

For the other term, $\delta_h \psi_m$, one has,

$$\gamma^{mn} \delta_h \psi_m = -\partial_r h_{ms} \gamma^{mn} \gamma^{rs} \epsilon.$$

Using the identity for the product $\gamma^{mn} \gamma^{rs}$ given above yields,

$$\begin{aligned} \gamma^{mn} \delta_h \psi_m &= (-\partial^n h_{ns} \gamma^{ms} + \gamma^{mr} \partial_r h - \partial^r h_n^m \gamma^{nr}) \epsilon \\ &\quad + (-\partial^n h_{mn} + \partial^m h) \epsilon. \end{aligned}$$

Expressing h_{ij} in terms of Φ_{ij} gives,

$$\gamma^{mn} \delta_h \psi_m = \epsilon^{kmp} \partial_k (\partial^r \Phi_{br} \gamma_p^b + 2\partial_b \Phi_{pc} \gamma^{cb} - \partial_b \Phi \gamma_p^b + \partial^n \Phi_{np}) \epsilon \quad (C.2)$$

This expression takes the form $\epsilon^{kmp}\partial_k\Xi_p$ for some Ξ_p and is at the same time equal to,

$$2\epsilon^{kmp}\gamma_5\partial_k\delta_h\chi_p, \quad (\text{C.3})$$

if one expresses the variation of ψ_m in terms of the variation of χ_p . Accordingly, by mere comparison of (C.3) with (C.2), one can read off the supersymmetry variation of the vector-spinor prepotential. Now, one can always add to $\delta\chi_p$ an arbitrarily chosen gauge transformation of χ_p . This is just a matter of definition and we will use this freedom to simplify the supersymmetry variation of χ_p .

It turns out that the first term and the last term in the right-hand side of (C.2) are just a gauge transformation of χ_p since

$$\begin{aligned} \partial^r\Phi_{br}\gamma_p{}^b + \partial^n\Phi_{np} &= -\frac{1}{2}\partial^r\Phi_{br}\gamma^b\gamma_p + \partial^n\Phi_{np} + \gamma_p A \\ &= -\partial^r\Phi_{br}\delta_p^b + \partial^n\Phi_{np} + \gamma_p A' \\ &= \gamma_p A' \end{aligned}$$

for some A, A' . Similarly, the third term in (C.2) is also a gauge transformation,

$$\partial_b\Phi\gamma_p{}^b = -\frac{1}{2}\partial^b\Phi\gamma^b\gamma_p + \gamma_p B = -\partial_p\Phi + \gamma_p B',$$

for certain B and B' , which yields also a gauge transformation of χ_p .

So we conclude that $2\gamma_5\delta_h\chi_p = 2\partial_b\Phi_{pc}\gamma^{cb}\epsilon$, i.e.,

$$\delta_h\chi_p = -\partial_b\Phi_{pc}\gamma^{cb}\gamma_5\epsilon. \quad (\text{C.4})$$

Adding (C.1) and (C.4) together yields finally,

$$\delta\chi_p = (\partial_b P_{pc} - \partial_b\Phi_{pc}\gamma_5)\gamma^{cb}\epsilon. \quad (\text{C.5})$$

C.2 Bosonic prepotential $Z_{mn}^2 \equiv \Phi_{mn}$

One has,

$$\delta h_{ij} = i\bar{\epsilon}\gamma_i\psi_j + (i \leftrightarrow j). \quad (\text{C.6})$$

Substituting the expression for ψ_j yields

$$\delta h_{ij} = i\bar{\epsilon}\gamma_i \left(-\gamma_5\partial^k\chi^p\epsilon_{kjp} + \gamma_{js}\gamma_5\partial_k\chi_p\epsilon^{ksp} \right) + (i \leftrightarrow j).$$

Using $\gamma_i \gamma_{js} = \delta_{ij} \gamma_s - \delta_{is} \gamma_j - \epsilon_{ijs} \gamma_0 \gamma_5$ and $\delta_{ij} \epsilon_{ksp} = \delta_{jk} \epsilon_{isp} + \delta_{js} \epsilon_{ipk} + \delta_{jp} \epsilon_{iks}$ one then gets,

$$\delta h_{ij} = i \epsilon_{jkp} \partial^k (\bar{\epsilon} \gamma_i \gamma_5 \chi^p + \bar{\epsilon} \gamma^p \gamma_5 \chi_i) + \partial_j (\bar{\epsilon} \gamma^s \gamma_5 \chi^p \epsilon_{isp}) + (i \leftrightarrow j).$$

This implies, for the prepotential Φ_{mn} and the linearized diffeomorphism u_m ,

$$\delta \Phi_{mn} = i (\bar{\epsilon} \gamma_m \gamma_5 \chi_n + \bar{\epsilon} \gamma_n \gamma_5 \chi_m), \quad (\text{C.7})$$

$$\delta u_m = i \bar{\epsilon} \gamma^s \gamma_5 \chi^p \epsilon_{msp}. \quad (\text{C.8})$$

C.3 Bosonic prepotential $Z_{mn}^1 \equiv P_{mn}$

The supersymmetry variation of the conjugate momentum π^{ij} is most easily obtained from the supersymmetry variation of the metric and the equations of motion.

One has,

$$\delta(\partial_0 h_{ij}) = i \bar{\epsilon} \gamma_i \partial_0 \psi_j + (i \leftrightarrow j),$$

and,

$$\delta(\partial_j h_{i0}) = i \bar{\epsilon} \gamma_i \partial_j \psi_0 + i \bar{\epsilon} \gamma_0 \partial_j \psi_i.$$

Using the equations of motion (B.3) for ψ_j and the identity for $\gamma_i \gamma_{jm}$ just recalled above yields,

$$\delta K_{ij} = \frac{i}{2} \bar{\epsilon} \gamma_0 \partial_j \psi_i + \frac{i}{2} \epsilon_{jrs} \bar{\epsilon} \gamma_i \gamma_5 \partial^r \psi^s - \frac{i}{4} \delta_{ij} \epsilon^{mrs} \bar{\epsilon} \gamma_m \gamma_5 \partial_r \psi_s + (i \leftrightarrow j).$$

This implies,

$$\delta \pi^{ij} = -\frac{i}{2} \bar{\epsilon} \gamma_0 \partial^i \psi^j - \frac{i}{2} \epsilon^{irs} \bar{\epsilon} \gamma^j \gamma_5 \partial_r \psi_s + \frac{i}{2} \delta^{ij} \bar{\epsilon} \gamma_0 \partial^k \psi_k + (i \leftrightarrow j),$$

for the conjugate momenta.

Now, we substitute in the variation of π^{ij} the expression (B.5) for ψ_m in terms of the spinor prepotential and expand the matrices occurring in the resulting equation in the basis $\{I, \gamma_\mu, \gamma_{\mu\nu}, \gamma_{\mu\nu} \gamma_5, \gamma_\mu \gamma_5, \gamma_5\}$ using the relevant identities. One expects a priori terms involving $\gamma_0 \gamma_5$ or γ_k . However, a direct computation shows that the terms containing $\gamma_0 \gamma_5$ actually cancel, and that

$\delta\pi^{ij}$ reduces to

$$\begin{aligned}\delta\pi^{ij} = & -\frac{i}{2}\bar{\epsilon}\gamma^a\partial^i\partial_k\chi_p\epsilon^j{}_{sa}\epsilon^{ksp} + (i \leftrightarrow j) \\ & +\frac{i}{2}\delta^{ij}\bar{\epsilon}\gamma^a\partial_m\partial_k\chi_p\epsilon^m{}_{sa}\epsilon^{ksp} + (i \leftrightarrow j) \\ & -\frac{i}{2}\bar{\epsilon}\gamma^j\partial_r\partial^k\chi_p\epsilon^{irs}\epsilon_{ksp} + (i \leftrightarrow j) \\ & +\frac{i}{2}\bar{\epsilon}\gamma_s\partial_r\partial_k\chi_p\epsilon^{irs}\epsilon^{jkp} + (i \leftrightarrow j).\end{aligned}$$

By expanding the products of ϵ^{bcd} 's, one easily verifies that the sum of the first three terms is equal to the last term, so that,

$$\delta\pi^{ij} = i\bar{\epsilon}\gamma_s\partial_r\partial_k\chi_p\epsilon^{irs}\epsilon^{jkp} + (i \leftrightarrow j),$$

an expression which we can rewrite as

$$\delta\pi^{ij} = i\epsilon^{irs}\epsilon^{jkp}\partial_r\partial_k(\bar{\epsilon}\gamma_s\chi_p + \bar{\epsilon}\gamma_p\chi_s).$$

The searched-for variation of the prepotential P_{mn} then follows,

$$\delta P_{mn} = i(\bar{\epsilon}\gamma_m\chi_n + \bar{\epsilon}\gamma_n\chi_m). \quad (\text{C.9})$$

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